

☒ Stress and Deformation Analysis

Material in this lecture was taken from chapter 3 of Most, Machine Elements in Mechanical Design, 2003

Representing Stresses on a Stress Element

- One main goal of stress analysis is to determine the point within a load-carrying member that is subjected to the highest stress level.
- The orientation of the stress element is critical, and it must be aligned with specified axes on the member, usually called x , y , and z .

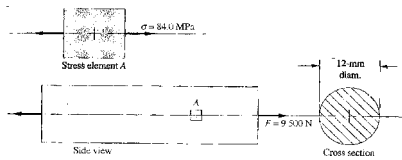
Representing Stresses on a Stress Element con't

- Tensile and compressive stresses, called normal stresses, act perpendicular to the opposite faces of the stress element.
- Tensile stresses pull on the element.
- Compressive stresses crush the element.

Direct Stresses: Tension and Compression

- Stress = the internal resistance offered by a unit area of a material to an externally applied load. Normal stresses (σ) can be tensile (positive) or compressive (negative).
- The magnitude of the stress can be calculated from the direct stress formula:
 - $\sigma = \text{force} / \text{area} = F/A$
 - (US Units) $\text{lb} / \text{in}^2 = \text{psi}$; (SI Units) $\text{N} / \text{m}^2 = \text{pascal} = \text{Pa}$

Example Problem 3-1 A tensile force of 9500 N is applied to a 12-mm-diameter round bar, as shown in Figure 3-6. Compute the direct tensile stress in the bar.



- Objective** Compute the tensile stress in the round bar.
- Given** Force = $F = 9500 \text{ N}$; diameter = $D = 12 \text{ mm}$.
- Analysis** Use the direct tensile stress formula, Equation (3-1): $\sigma = F/A$. Compute the cross-sectional area from $A = \pi D^2/4$.
- Results** $A = \pi D^2/4 = \pi(12 \text{ mm})^2/4 = 113 \text{ mm}^2$
 $\sigma = F/A = (9500 \text{ N})/(113 \text{ mm}^2) = 84.0 \text{ N/mm}^2 = 84.0 \text{ MPa}$
- Comment** The results are shown on stress element A in Figure 3-6, which can be taken to be anywhere within the bar because, ideally, the stress is uniform on any cross section. The cube form of the element is as shown in Figure 3-5 (a).

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Direct Stresses: Tension and Compression con't

- The conditions on the use of the previous equation are as follows:
 1. The load-carrying member must be straight
 2. The line of action of the load must pass through the centroid of the cross-section of the member
 3. The member must be of uniform cross section near where the stress is being computed
 4. The material must be homogeneous and isotropic
 5. In the case of compression members, the member must be short to prevent buckling.

Deformation Under Direct Axial Loading

The following formula computes the stretch due to a direct axial tensile load or the shortening due to a direct axial compressive load:

$$\delta = FL/EA \quad (3-2)$$

where δ = total deformation of the member carrying the axial load

F = direct axial load

L = original total length of the member

E = modulus of elasticity of the material

A = cross-sectional area of the member

Noting that $\sigma = F/A$, we can also compute the deformation from

$$\delta = \sigma L/E \quad (3-3)$$

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Deformation Under Direct Axial Loading

Stretch due to axial tension, or shortening due to axial compression

$$\delta = \frac{F * L}{E * A}$$

Where * = total deformation

F = direct axial load

L = Original length

E = modulus of elasticity

A = cross-sectional area

Noting that stress $F = F/A$, we get * = $F * L / F$

Direct Shear Stress

- Direct shear stress occurs when the applied force cuts through the member like scissors, or when a punch and a die are used to punch a slug of material from a sheet.
- The method of computing direct shear stress is similar to method used for computing direct tensile stress. Both methods assume the applied force is uniformly distributed across the cross section of the part that is resisting the force.
- τ = shearing force / area in shear = F / A_s

Relationship Among Torque, Power, and Rotational Speed

- The relationship among the power (P), the rotational speed (n), and the torque (T) in a shaft is described by the equation:
 - $T = P / n$ (SI) or $T = 63,000 P / n$ (English)
- In SI units, power is expressed in:
 - watt (W) or its equivalent
 - newton meter per second (N*m/s)
- Rotational speed is in radians per second (rad/s)

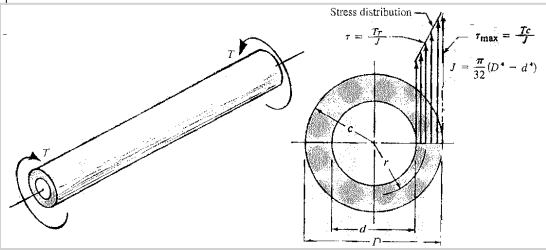
Torsional Shear Stress

- A torque causes a rotation of one part of the member relative to another, which leads to shear stress.
- In torsional shear, the distribution of stress is not uniform across the cross section.
 - Most common case: a round circular shaft transmitting power.

Torsional Shear Stress Formula

- When the outer surface of a solid round shaft is subjected to torque, it experiences the greatest shearing strain and the largest torsional shear stress.
 - $\tau_{\max} = T \cdot c / J$
 - Where c = radius of the shaft to its outside surface
 - J = polar moment of inertia

Stress Distribution in a Hollow Shaft



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Polar Section Modulus

A design simplification uses the polar section modulus,

- $Z_p = J / c$
- The equation for the maximum torsional shear stress is:
 - $\tau_{\max} = T / Z_p$
- When one cross-section is rotated relative to other cross-sections due to torque, the angle of twist is computed from:
 - $\theta = TL / GJ$
 - Where θ = angle of twist (radians)
 - L = length of the shaft over which the angle is being computed
 - G = modulus of elasticity of the shaft material in shear

Vertical Shearing Stress

- A beam carrying loads transverse to its axis will experience shearing forces (V).
- To analyze the beams, you compute the variation in shearing force across the entire length of the beam and draw the shearing force diagram.
- The resulting vertical shearing stress τ is computed by:
 - $\tau = VQ / It$
 - Where I = rectangular moment of inertia of the cross section of the beam
 - t = thickness of the section at the place where the shearing stress is to be computed
 - Q = first moment, with respect to the overall centroidal axis

Results $I = th^3/12 = (2.0 \text{ in})(8.0 \text{ in})^3/12 = 85.3 \text{ in}^4$
 $Q = A_p \bar{y} = th^2/8 = (2.0 \text{ in})(8.0 \text{ in})^2/8 = 16.0 \text{ in}^3$

Then the maximum shearing stress is

$$\tau = \frac{VQ}{It} = \frac{(1000 \text{ lb})(16.0 \text{ in}^3)}{(85.3 \text{ in}^4)(2.0 \text{ in})} = 93.8 \text{ lb/in}^2 = 93.8 \text{ psi}$$

Comments The maximum shearing stress of 93.8 psi occurs at the neutral axis of the rectangular section as shown in Figure 3-14. The stress distribution within the cross section is generally parabolic, ending with zero shearing stress at the top and bottom surfaces. This is the nature of the shearing stress everywhere between the left support at A and the point of application of the 1200-lb load at B. The maximum shearing stress at any other point in the beam is proportional to the magnitude of the vertical shearing force at the point of interest.

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Special Shearing Stress Formulas

- The most common cross sections have unique formulas for finding the maximum vertical shearing stress:

$$\tau_{\max} = 3V/2A \quad (\text{for rectangle})$$

$$\tau_{\max} = 4V/3A \quad (\text{for circle})$$

Stress Due to Bending

- A beam is a member that carries loads transverse to its axis, producing bending moments in the beam. Bending stresses are normal stresses (tensile or compressive).
- The maximum bending stress in a beam cross section occurs in the part farthest from the neutral axis of the section.

$$\sigma = M*c / I$$

Where M = magnitude of bending moment at section
 I = moment of inertia of section w.r.t neutral axis
 c = distance from n.a. to outer most fiber

Stress Due to Bending con't

- The value varies from zero at the neutral axis, to the maximum tensile stress on one side of the neutral axis, to the maximum compressive stress on the other side.
- Positive bending only occurs when the deflected shape of the beam is concave upward, resulting in compression on the upper part of the cross section and tension on the lower part.
- Negative bending causes the beam to be concave downward.

Stress Due to Bending con't

- The flexure formula was developed subject to the following conditions:
 1. The beam must be in pure bending. Shearing stresses must be zero or negligible. No axial loads present.
 2. The beam must not twist or be subjected to a torsional load.
 3. The material of the beam must obey Hooke's law.
 4. The modulus of elasticity of the material must be the same in both tension and compression.

Stress Due to Bending con't

5. The beam is initially straight and has a constant cross section
6. Any plane cross section of the beam remains plane during bending.
7. No part of the beam shape fails because of local buckling or wrinkling.

Combined Normal Stresses

- When the same cross section of a load-carrying member experiences both a direct normal stress and a bending stress, the resulting normal stress can be computed by the method of superposition.

- $\sigma = \pm (Mc / I) \pm (F / A)$

- Where tensile stresses are positive and compressive stresses are negative.

Combined Normal Stresses con't

- An example of this situation involves a beam subjected to a load applied downward and to the right through a bracket below the beam. The load's horizontal and vertical components show that the effect can be broken into three parts:
 1. The vertical component places the beam in bending with tension on the top and compression in the bottom.
 2. The horizontal component, because it acts away from the neutral axis of the beam, causes bending with tension on the bottom and compression on the top.
 3. The horizontal component causes direct tensile stress across the entire cross section.

Combined Normal Stresses con't

1. Prepare the shearing force and bending moment diagrams.
2. Combine the effects of the bending stress and the direct tensile stress at any point.

